

# PORTFOLIO MANAGEMENT

## CLASS 4

### HOME WORK SUPPORT

### COVERAGE

Question			Answer			Lecture Time
Q. No	Page no.	Book	Q. No	Page no.	Book	
1	124	HW ANS BOOK	1	125	HW ANS BOOK	00:00:33 TO 00:02:15
2	126	HW ANS BOOK	2	126	HW ANS BOOK	00:02:16 TO 00:03:15
1	127	HW ANS BOOK	1	127	HW ANS BOOK	00:03:16 TO 00:06:26
14	49	CW Q BOOK	14	89	CW ANS BOOK	00:06:27 TO 00:17:17

**Topic 5** MINIMUM RISK PORTFOLIO AND RISK FREE PORTFOLIO

**Question 1:** SSEI HW ANSWER BOOK Q1 PAGE 124

An investor has decided to invest ₹ 1,00,000 in the shares of two companies, namely, ABC and XYZ. The projections of returns from the shares of the two companies along with their probabilities are as follows:

Probability	ABC(%)	XYZ(%)
30	18	08
20	14	11
40	17	05
10	10	16

You are required to:

- i. Comment on return and risk of investment in individual shares.
- ii. Compare the risk and return of these two shares with a Portfolio of these shares in equal proportions.
- iii. Find out the proportion of each of the above shares to formulate a minimum risk portfolio.

(Source: FOD)

**ANSWER:** SSEI HW ANSWER BOOK Q1 PAGE 125

i.

Probability	ABC (%)	XYZ (%)	1X2 (%)	1X3 (%)
(1)	(2)	(3)	(4)	(5)
0.30	18	08	5.40	2.40
0.20	14	11	2.80	2.20
0.40	17	05	6.80	2.00
0.10	10	16	1.00	1.60
<b>Average return</b>			<b>16</b>	<b>8.20</b>

Hence the expected return from ABC = 16% and XYZ is 8.20%

Probability	$(ABC - \overline{ABC})$	$(ABC - \overline{ABC})^2$	1X3	$(XYZ - \overline{XYZ})$	$(XYZ - \overline{XYZ})^2$	(1)X(6)
(1)	(2)	(3)	(4)	(5)	(6)	
0.30	2	4	1.20	-0.20	0.04	0.012
0.20	-2	4	0.80	2.80	7.84	1.568
0.40	1	1	0.40	-3.20	10.24	4.096
0.10	-6	36	3.60	7.80	60.84	6.084
			<b>6.00</b>			<b>11.76</b>

Sd of ABC = 2.45%

Sd of XYZ = 3.43%

ii. In order to find risk of portfolio of two shares, the covariance between the two is necessary here.

Probability	$(ABC - \overline{ABC})$	$(XYZ - \overline{XYZ})$	2X3	1X4
(1)	(2)	(3)	(4)	(5)
0.30	2	-0.20	-0.40	-0.12
0.20	-2	2.80	-5.60	-1.12
0.40	1	-3.20	-3.20	-1.28
0.10	-6	7.80	-46.80	-4.68
				<b>-7.20</b>

$$\sigma_p^2 = (0.50 \times 2.45)^2 + (0.50 \times 3.43)^2 + 2 \times 0.5 \times 0.5 \times (-7.20)$$

$$\sigma_p^2 = 0.84185$$

$$\sigma_p = 0.92\%$$

$$E(R_p) = (0.5 \times 16) + (0.5 \times 8.2) = 12.1\%$$

Hence, the return is 12.1% with the risk of 0.92% for the portfolio. Thus the portfolio results in the reduction of risk by the combination of two shares.

iii. For constructing the minimum risk portfolio the condition to be satisfied is

$$X_{ABC} = \frac{\sigma_X^2 - r_{AX} \sigma_A \sigma_X}{\sigma_A^2 + \sigma_X^2 - 2r_{AX} \sigma_A \sigma_X} \text{ or } = \frac{\sigma_X^2 - \text{Cov.}AX}{\sigma_A^2 + \sigma_X^2 - 2\text{Cov.}AX}$$

$\sigma_X$  = Std. Deviation of XYZ

$\sigma_A$  = Std. Deviation of ABC

$r_{AX}$  = Coefficient of Correlation between XYZ and ABC

$\text{Cov.}AX$  = Covariance between XYZ and ABC.

Therefore,

$$\% \text{ ABC} = \frac{11.76 - (-7.20)}{6.00 + 11.76 - [2(-7.20)]}$$

$$\% \text{ ABC} = 59\%, \text{ XYZ} = 41\%$$

**Question 2:** SSEI HW ANSWER BOOK Q2 PAGE 126

The following information about two stocks are given

	Stock A	Stock B
Return	15%	18%
Risk	9%	12%

Correlation coefficient between the two stocks is -1

Construct a risk-free portfolio and calculate the return and risk of that portfolio

(Source: FOD)

**ANSWER:** SSEI HW ANSWER BOOK Q2 PAGE 126

To construct a risk-free portfolio first we need to compute the weights of both the stocks

$$\text{Weight of A} = \frac{SD\ OF\ B}{SD\ OF\ A + SD\ OF\ B} = 12 / (9 + 12) = 57.14\%$$

$$\text{Weight of B} = 1 - \text{weight of A} = 42.86\%$$

$$\text{Return of the portfolio} = \text{weighted average} = (0.5714 \times 15) + (0.4286 \times 18) = 16.23\%$$

$$\begin{aligned} \text{Variance of the portfolio} &= (0.5714 \times 9)^2 + (0.4286 \times 12)^2 + 2 \times 0.5714 \times 0.4286 \times 9 \times 12 \times (-1) \\ &= 0\% \end{aligned}$$

Risk of the portfolio = 0% i.e. risk-free portfolio

**Topic 6** N STOCK WORLD

**Question 1:** SSEI HW ANSWER BOOK Q1 PAGE 127

Suppose that in the universe of available risky securities contains a large number of shares two stocks, identically distributed with  $E(r) = 18\%$ , and  $\sigma = 25\%$ , and with a common correlation coefficient of  $\rho = 0.4$

- What is the expected return and standard deviation of an equally weighted risky portfolio of 40 stocks?
- What is the smallest number of stocks necessary to generate an efficient portfolio with a standard deviation equal to or smaller than 16%?
- What is the systematic risk in this security universe?
- If T-bills are available and yield 8%, what is the slope of the CAL?

(Source: ICAI)

**ANSWER:** SSEI HW ANSWER BOOK Q1 PAGE 127

The parameters are  $E(R) = 18$ ,  $\sigma = 25$ , and the correlation between any pair of stocks is  $\rho = 0.40$

- a. The portfolio expected return is invariant to the size of the portfolio because all stocks have identical expected returns. The standard deviation of a portfolio with  $n = 40$  stock is

$$\begin{aligned}\sigma_p &= \left[ \sigma^2/n + \rho \times \sigma^2(n-1)/n \right]^{1/2} \\ &= \left[ \frac{625}{40} + 0.40 \times 625 \times \frac{39}{40} \right]^{1/2} = 16.10\end{aligned}$$

- b. Because the stocks are identical, efficient portfolios are equally weighted. To obtain a standard deviation of 16%, we need to solve for  $n$ :

$$\begin{aligned}16 &= \left[ \frac{625}{n} + 0.4 \times 625 \times \left( 1 - \frac{1}{n} \right) \right]^{0.5} \\ n &= 375/6 = 62.5\end{aligned}$$

Thus we need 63 stock and will come in with volatility slightly under the target.

- c. As  $n$  gets very large, the variance of an efficient (equally weighted) portfolio diminishes, leaving only the variance that comes from the covariances among stocks, that is

$$\sigma_p = \sqrt{\rho \times \sigma^2} = (0.40 \times 625)^{0.50} = 15.81$$

- d. If the risk-free is 8%, then the risk premium on any size portfolio is  $18\% - 8\% = 10\%$ . The standard deviation of a well-diversified portfolio is (practically) 15.81%; hence the slope of the Capital Allocation Line (CAL) is  
 $S = 10/15.81 = 0.6325$

**Question 14:** SSEI CW QUESTION BOOK Q14 PAGE 49

Suppose that in the universe of available risky securities contains a large number of shares and stocks, identically distributed with  $E(r) = 15\%$ , or  $\sigma = 60\%$ , and with a common correlation coefficient of  $\rho = 0.5$ .

- What is the expected return and standard deviation of an equally weighted risky portfolio of 25 stocks?
- What is the smallest number of stocks necessary to generate an efficient portfolio with a standard deviation equal to or smaller than 43%?
- What is the systematic risk in this security universe?
- If T-bills are available and yield 10%, what is the slope of the CAL?

**(Source: ICAI)**

**Answer:** SSEI CW ANSWER BOOK Q14 PAGE 89

The parameters are  $E(R) = 15$ ,  $\sigma = 60$ , and the correlation between any pair of stocks is  $\rho = .5$ .

- The portfolio expected return is invariant to the size of the portfolio because all stocks have identical expected returns. The standard deviation of a portfolio with  $n = 25$  stock is

$$\begin{aligned} \sigma_p &= \left[ \sigma^2/n + \rho \times \sigma^2(n - 1)/n \right]^{1/2} \\ &= \left[ 60^2/25 + 0.5 \times 60^2 \times 24/25 \right]^{1/2} = 43.27 \end{aligned}$$

- Because the stocks are identical, efficient portfolios are equally weighted. To obtain a standard deviation of 43%, we need to solve for  $n$ :

$$43^2 = \frac{60^2}{n} + 0.5 \times \frac{60^2(n - 1)}{n}$$

$$1,849n = 3,600 + 1,800n - 1,800$$

$$n = 1,800/49 = 36.73$$

Thus we need 37 stock and will come in with volatility slightly under the target.

- As  $n$  gets very large, the variance of an efficient (equally weighted) portfolio diminishes, leaving only the variance that comes from the covariances among stocks, that is

$$\sigma_p = \sqrt{\rho \times \sigma^2} = \sqrt{0.5 \times 60^2} = 42.43$$

- If the risk-free is 10%, then the risk premium on any size portfolio is  $15\% - 10\% = 5\%$ . The standard deviation of a well-diversified portfolio is (practically) 42.43%; hence the slope of the Capital Allocation Line (CAL) is

$$S = 5/42.43 = 0.1178$$